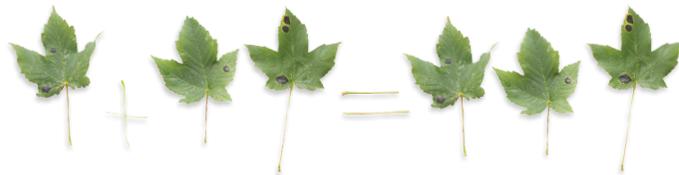


MESSY MATHS

A Playful, Outdoor
Approach for Early Years



JULIET ROBERTSON

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This book is dedicated to my sister, Sophie,
who loved maths.

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Most of my class are very confident at climbing trees. One day I had a new little girl who was not so sure. She stood and watched the other children for several minutes before one of her friends shouted for her to come up the tree. 'Come on, Alex. It's good up here. It is so high and you can see really far to the other side.' Another child added, 'Alex, you look tiny down there. Climb up and you will be big again. Miss Dean will be really small if you climb up.'

Alex continued to watch the others, remarking, 'Miss Dean, it's too scary up high.' One child, Rylan, said, 'It's okay, I can come down and help.' Rylan climbed down from the tree and went over to Alex. After a little bit of encouragement, Alex began to cautiously climb the tree. Rylan directed her using a host of positional language: 'Put your foot a little higher' and 'Move to the side by the fat branch.'

I was able to expand and support this language throughout the experience, and each time Alex got a little higher you could see her confidence growing more and more. When she had got as high as she could, we talked about what she could see and compared this to being on the ground.

When her mum came to pick her up, Alex was excited to tell her what she had done that day. She described her experience using the positional and size related language she had been exposed to. It was a fabulous learning experience for Alex, Rylan and me.

Louise Dean, early years teacher,
Redgate Community Primary School, Formby, Liverpool

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My thanks go to the following schools, nurseries and gardens whose children and grounds are featured in this book: Darton Primary; Ichijima-Kodomoen Nursery School; Hoyland Common Primary; Adelaide Botanic Garden; Cruickshank Botanic Garden; Kate Greenaway Nursery, Islington; Penguin District School; Little Acorns Day Nursery; Fraserburgh South Park School; Tullos Primary School Nursery; Wellington College, Shanghai and Worsbrough Common Primary.

In Chapters 9 and 13, the inspiration behind viewing the outdoor space mathematically is the work of Tom Bedard and his blog, Sand and Water Tables: <http://tomsensori.blogspot.co.uk>. He is a creator and connector of spaces.

Special thanks go to Amy Sinclair, founder of Active Learning in Maths, for her advice and support.

For more information about her work and courses visit: <http://www.activelearninginmaths.co.uk>.

Introduction

The world is a mathematical place. For young children who are naturally curious there are shapes, numbers, moving objects and patterns to behold, things to count and investigations to be undertaken. It is a creative, challenging and wondrous way of looking at life and exploring the world.

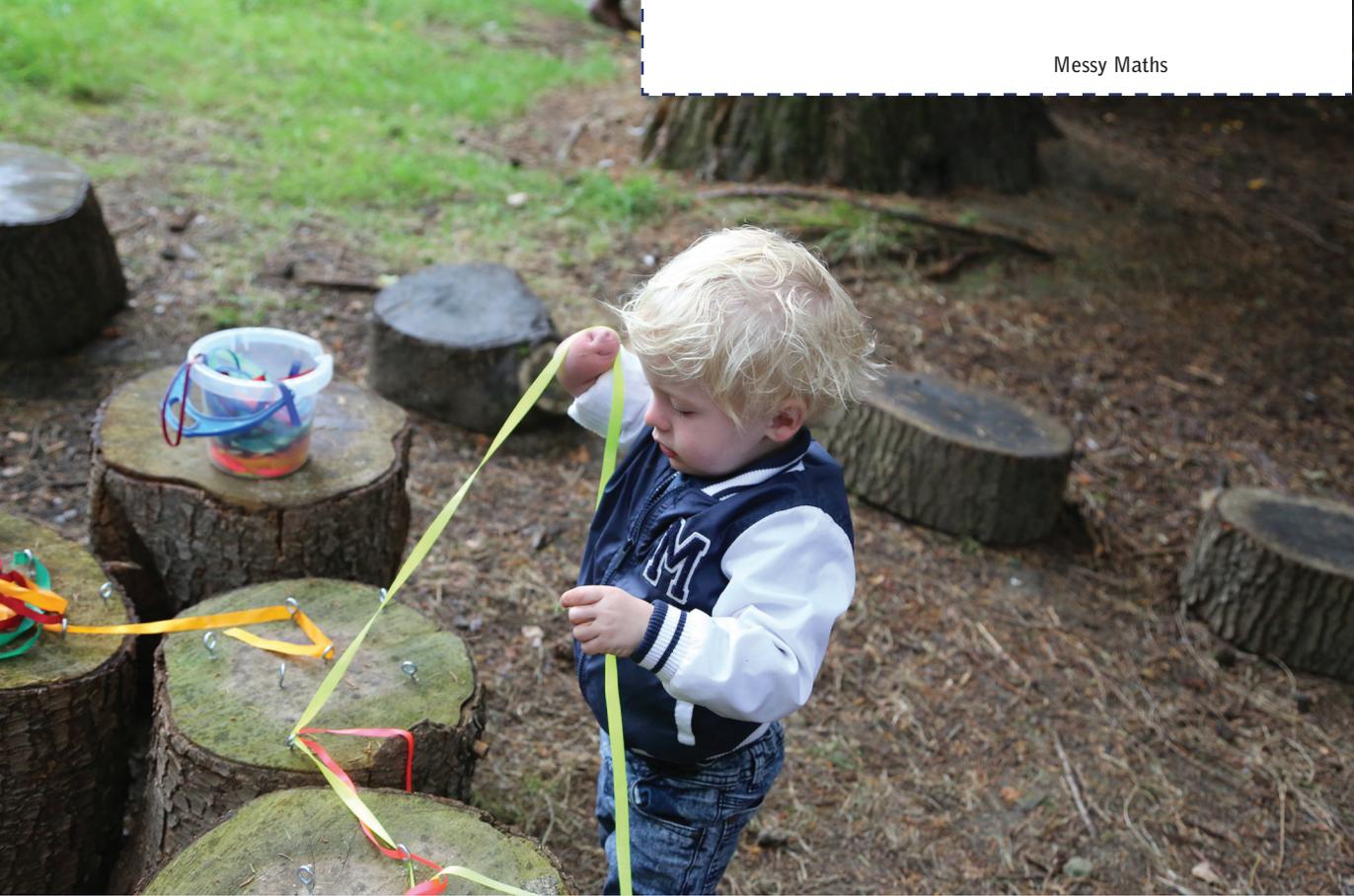
Maths is an inherent part of being human. Framing the world through maths helps us make sense of what is happening and how it affects us. It can provide order and certainty as well as help to explain random events. For little children, maths is not just a cognitive process. It is also a social, emotional and physical experience. A problem shared is a problem halved. Think back to your own experiences of maths at school and how you felt about learning it. Consider how children need lots of physical experiences that embody what maths is all about. The only way to understand the concept of weight, for example, is to repeatedly lift, move and carry a range of heavy and light objects.

Being outside enables connections to be made between the hands, heart and head, laying the foundations for more complex work as children grow, develop and learn. The natural and built worlds provide dynamic and constantly changing environments, offering an endless supply of patterns, textures, colours, quantities and other attributes that underpin much of the necessary early maths experiences.

This book has several aims:

- To help educators consider their own understanding and confidence in developing mathematical provision, making the most of the outdoor space in their setting – whether this be a nursery, playgroup, child-minder's back garden or a nature kindergarten.
- To enable children to have lots of informal, spontaneous mathematical experiences which are child initiated and child led. When children play, they are in control of their learning and it has meaning and purpose to them.
- To provide a range of open-ended ideas for adapting to children's interests as part of a playful approach to learning maths concepts. This is about educators being responsive to the needs and interests of children.
- To make the most of any outdoor space as a context for maths. We need to consider how time in greenspaces such as woodlands, parks and beaches contributes to children's mathematical experiences, as can the use of natural materials.

I believe that every child and adult is mathematically able. We all have different strengths and abilities within maths. By sharing ideas and enjoying mathematical conversations, we can deepen our understanding. For children who require additional support, we need to be sensitive to their



needs. Use language and communication methods appropriate to their abilities, have fun and focus on precisely what each child can do. They may surprise you and extend your learning just as much as you do theirs.

Being outside makes maths feel real rather than disconnected from our daily lives. And real maths is really messy! Aside from mud pies and puddles, the cognitive processes involved mean it is not a smooth linear pathway of learning but rather an interconnected network. Children need time to make sense of abstract mathematical ideas through experiential processes, along with opportunities to ponder, enjoy and discuss the concepts encountered. Lots of playing along the way is a must.

How to use this book

Messy Maths has been written for educators who work with children aged between approximately three and six years of age. It is not a 'how to' guide, rather a handy reference book: a source of ideas about tapping into the outdoors to help children become confident and skilled in thinking about, using and exploring mathematical concepts as they play outside. Many aspects of any early years maths curriculum can be addressed without the need for structured activities, but through

developing the four R's of routines, resources, the responsibilities of adults and reimagining the environment. This means looking at our outdoor space with a mathematical lens and considering the possibilities. This is discussed in Chapter 13.

Suggestions for integrating maths into routines have been amalgamated in Chapter 12. The intention is to provide explicit examples of possibilities to help develop creative, progressive and flexible approaches to embedding maths.

Throughout the book I refer to 'gathering circles.' These are times when the children have gathered together and are ready for, and interested in, undertaking a shared event. Sometimes gathering circles happen naturally outside, such as when lunch is shared together in a wood. More often, they are informal occurrences when children playing together congregate to view an exciting discovery or to simply relax and chat to each other.

A positive approach to being outside

Throughout the book, there are lots of ideas and accompanying photos that contain elements of risk. In line with best practice, it is important that you consider all the suggestions on a case-by-case basis to determine whether they are appropriate for the developmental stage and learning needs of your children. The adults who work with your children need to have the competence and confidence to ensure the routines, resources and environment are as safe as is necessary. Remember to undertake risk–benefit assessments, in line with your establishment's health and safety policies and procedures, for anything you feel needs it, be this using tools and ropes, exploring the mathematics of fire, climbing trees and other experiences involving height, moving heavy objects, working off-site or near water, and so on. *Messy Maths* is about enabling outdoor practice but every educator and setting needs to undertake this within a framework of safety.

Likewise, it is important that wherever you are working you follow the land access laws of your country. Being respectful of others, leaving no trace of your presence and considering the impact of your practice on the environment are all part of our responsibilities as educators.

The basics

The core building blocks of maths matter as much outside as they do indoors. Derek Haylock and Anne Cockburn propose that a 'mathematical concept can be thought of as a network of connections between symbols, language, concrete experiences and pictures.'¹ This means children need to:

- Recognise the symbols used in maths.
- Know and understand the language used.

1 D. Haylock and A. Cockburn, *Understanding Mathematics for Young Children: A Guide for Teachers of Children 3–8* (4th edn) (London: SAGE Publications, 2013), p. 27.

- Be able to create a mental or actual image of the concept.
- Have concrete experiences and relevant contexts for working in maths.

Most difficulties arise when not all of these elements are experienced or if they are not connected in a meaningful way. Undertaking maths activities outside provides an ideal environment for these connections to happen. It allows for skills and concepts learnt in one context to be readily applied to another.

Ann Montague-Smith and Alison Price replace concrete experiences with more specific detail.² They suggest that two important elements of maths are ‘physical materials’ and ‘real world scripts’. They state that real world tasks matter but also cite the value of ‘script’ in songs, rhymes and stories. These scripts provide children with relevant language and enable them to act out and retell stories that use the language in context. Daniela O’Neill, Michelle Pearce and Jennifer Pick studied the relationship between children’s narrative abilities in pre-school and found a predictive relationship with their later mathematical ability.³ For me, this is about enabling emotions to be expressed and time being made for children to make sense of concepts through role play, creative work and imaginative play. It is about employing strategies which engage children.

The other work which has influenced my thinking has been that of Czech mathematician Milan Hejný. His approach to maths teaching and child development uses twelve principles.⁴ These are grounded in common sense. The first principle is ‘building schemata’. Hejný suggests that children need to construct a network of mental mathematical schemata based upon their real life experiences – that is, to understand something you need to be able to connect the concept or idea to previous experiences. The Hejný approach uses experiences familiar to young children (such as walking) as a starting point to develop maths concepts and mental patterns and images.

The use of sticks is another example of a schema advocated by Hejný. Many children are familiar with and use sticks in their play, so the use of a stick to explain and illustrate mathematical thinking is building upon a known item. It is easy to make the link between a stick on the ground and a line drawn on a piece of paper. Thus the connection between using symbols and two-dimensional images is facilitated.

Finally, the concept of embodied cognition is particularly relevant in the context of how children grow and develop their mathematical understanding. This is about the acquisition of maths concepts being grounded in body movements and through interactions with the environment. Think about how children use their fingers as they learn to count. Hortensia Soto-Johnson suggests that highlighting students’ gestures, repeating their words, mimicking their gestures and asking

2 A. Montague-Smith and A. Price, *Mathematics in Early Years Education* (3rd edn) (Abingdon: Routledge, 2012), p. 16.

3 D. K. O’Neill, M. J. Pearce and J. L. Pick, Predictive relations between aspects of preschool children’s narratives and performance on the Peabody Individualized Achievement Test – Revised: evidence of a relation between early narrative and later mathematical ability. *First Language*, 24 (2004): 149–183.

4 See: <http://www.h-mat.cz/en/principles>.



probing questions can all help them to articulate their thinking.⁵ By paying attention to gesture, educators can learn more about children's mathematical reasoning and misconceptions.

One metre challenge

This activity is useful to undertake with parents or colleagues. In an outdoor space, ask everyone to create a line that is exactly one metre long using natural materials they can find.

Afterwards, discuss how each person worked out their metre. Provide measuring tapes or metre sticks so the results can be checked. Whilst there is always variation, most people rely on their previous knowledge about the approximate length of a metre. Some people relate it to a part of their body, others use a known reference such as the number of times an A4 sheet of paper fits into a metre. This 'knowing' is what children need to acquire through lots of practical play-based explorations of maths.

⁵ H. Soto-Johnson, Learning mathematics through embodied activities. *American Mathematical Society* (8 February 2016). Available at: <http://blogs.ams.org/matheducation/2016/02/08/learning-mathematics-through-embodied-activities/#sthash.LeIk8Lne.dpbs>.

Outside, ensuring children have plenty of access to natural environments where they can develop physically through movement and interactions with nature is particularly beneficial. The inextricable link between the development of the brain and body is summed up nicely by Jan White: ‘Movement and action give children intuitive, bodily-felt meanings about concepts that will later be understood more intellectually (such as many of the ideas we use in maths like weight and size). Embodied experiences also create deeper, more lasting memories which can be drawn on in different ways.’⁶ In other words, children need to use their whole bodies to learn maths.

The educator’s role

The role of adults is flagged up repeatedly in books and research articles. Pradnya Patet provides an eloquent summary of how educators can empower young mathematical minds.⁷ She argues that mathematical proficiency does not appear on its own but needs careful scaffolding that is meaningful and relevant. When children play you can see their level of understanding in a genuine context.

As educators we can facilitate maths play outdoors in many ways:

- We can set up outdoor areas in mathematical ways and embed maths into our routines.
- When redesigning an outdoor space, we can take account of features that can facilitate mathematical dialogue, explorations and investigations (see Chapter 13).
- We must make the most of teachable moments to introduce the language and specific skills that enhance children’s ability to reason mathematically. We should also offer suggestions, statements and challenges which encourage children to articulate their thinking. We want children to ask questions, discuss problems that arise and to not fear making mistakes. Errors are crucial to learning.
- We can be ready to build upon what children are doing through their play, their interests and what they like doing outside. The possible lines of development can be a mix of resources, investigations and simple prompts that support independent mathematical play.
- We can ensure that children themselves are the overwhelming evidence of their own learning and achievements in maths, rather than focusing on a paper-dependent system.

6 J. White, *Every Child A Mover: A Practical Guide to Providing Children with the Physical Opportunities They Need* (London: Early Education, 2015), p. 16.

7 P. Patet, Empowering mathematical minds through play. *Community Playthings* (8 September 2015). Available at: <http://www.communityplaythings.co.uk/learning-library/articles/empowering-mathematical-minds>.



Modelling a mathematical mindset

How adults respond to children regarding maths really counts. Jo Boaler's book, *Mathematical Mindsets*, advocates practical strategies for ensuring that the adults who work with children adopt a growth mindset approach that makes maths enjoyable and achievable.⁸

Everyone can achieve in maths, and we are always improving our knowledge and understanding. I believe a key benefit of outdoor play is that we can build confidence in maths concepts before children even perceive their play as mathematical, and in doing so we help to lessen, challenge or even prevent the development of negative connotations about the subject.

Practitioners need to actively show an interest in, and enthusiasm for, maths. Modelling maths activities, showing children how to use different resources, using mathematical language and being up for having fun with maths makes a big difference. Pass on a love of numbers.

⁸ J. Boaler, *Mathematical Mindsets: Unleashing Students' Potential Through Creative Math, Inspiring Messages and Innovative Teaching* (San Francisco, CA: Jossey-Bass, 2016).



Where's the Maths In That?

The purpose of this chapter is to provide some starting points for reflecting upon the mathematics which is inherent in children's play. We work in an educational culture in which the pressure is on us to be continually interacting, extending children's learning through dialogue, modelling what is expected or leading a structured activity. The best thing we can do is stop, step back, observe what the children are doing and reflect upon the maths we are witnessing.

By unpicking what we see children doing from a mathematical perspective, it is easier to consider the resources, experiences and conversations that are required to develop the skills, knowledge and understanding they need to gain next. This chapter includes several exercises to help you do this and considers some of the issues facing educators when providing maths opportunities outside.

Idea 1.1 Ensuring national expectations are achieved

In many countries there are statements about what children need to know and be able to do in order to reach a desired standard at a given age. Educators are required to make assessments and use their professional judgement to ensure that every child achieves what is expected of them. A range of evidence is needed. Within early years settings, most of this evidence can be collated through observations of and discussions with children. Also, there are lots of games and fun ways of eliciting a child's level of attainment.

Children need a balanced education that takes account of their social, emotional, physical and cognitive development. There is a lot of evidence which all points towards outdoor free play in a natural setting as providing the optimal conditions for this to happen, when combined with sensitive, nurturing adult support.¹ Being outside means children can apply their knowledge and skills across a range of real life contexts, thereby consolidating their understanding.

To isolate elements of maths without considering this wider context is not giving either the child or the subject an opportunity to flourish. Curriculum guidelines always advocate an embedded approach. It is up to those of us who work with children to interpret this advice in a holistic way that matches best practice and research. A standardised maths test is only one small part of the bigger picture. It will not be able to ascertain a child's ability to use maths for real. A couple of years after I began teaching, I met a five-year-old pupil on her own in a local shop. She told me with delight that she would have 72 pence change from one pound after she had bought her Mars

¹ The Children & Nature Network features a comprehensive collection of research which is curated and updated at: <http://www.childrenandnature.org/learn/research/>.

bar. As a young teacher, I realised that my focus on national standards had limited my own expectations about what this child could do.

Try to avoid getting hung up on evidencing what you have planned and provided. Taking photos of and writing lengthy documentation about what you have covered in a day or week is no indication of what the children have learnt. Shift the focus from what you are doing as a teacher to what your children know and demonstrate in their daily lives at home and in school.

Idea 1.2 Looking through a mathematical lens



Anything we do can be described from a mathematical perspective. It can enrich and help us to understand our world a little better. Imagine a child climbing a tree. There may be little to suggest that they are doing anything mathematical in this act. Yet if we stop and think, there is quite a bit of maths involved:

- Constant estimation: will this branch be strong enough to support my weight? Can I reach the next branch?

REAL MATHS IS REALLY MESSY

Children need physical experiences to make sense of the abstract ideas of maths and to ponder, enjoy and discuss the concepts encountered.

In *Messy Maths* Juliet Robertson offers a rich resource of ideas that will enable you to tap into the endless supply of patterns, textures, colours and quantities of the outdoors and deepen children's understanding through hands-on experience.

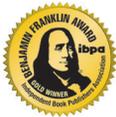
Packed full of activities that reimagine the outdoor space through a mathematical lens, this book is suitable for all early years educators (of ages 3–6) who want to shake up their usual classroom routine and adopt a playful and child-led approach to maths.

A wonderful resource! Aesthetically beautiful with its engaging photographs, it is easy and inspiring to read and offers meaningful and practical ideas for exploring maths outside of the classroom.

Mairi Ferris, Director, Stramash Outdoor Nurseries

Wow, what a fabulous book! Incredibly comprehensive and full of exciting possibilities, it should convince anyone of the huge potential of the outdoors.

Jan White, mudologist and consultant for outdoor provision in the early years



2018 IBPA Benjamin Franklin Awards™ gold winner, education category



2018 Education Resources Awards finalist, Educational Book Award

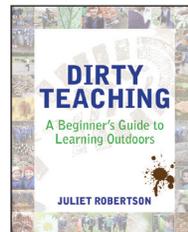


2017 Foreword INDIES Book of the Year Awards finalist, education category



Juliet Robertson is an education consultant who specialises in outdoor learning. Before becoming a consultant Juliet was a head teacher at three schools, making her more than qualified to help others improve their practice. She also writes a popular education blog – ‘I’m a teacher, get me OUTSIDE here!’ – where she illustrates her ideas and enthusiasm for learning outdoors. @CreativeSTAR

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