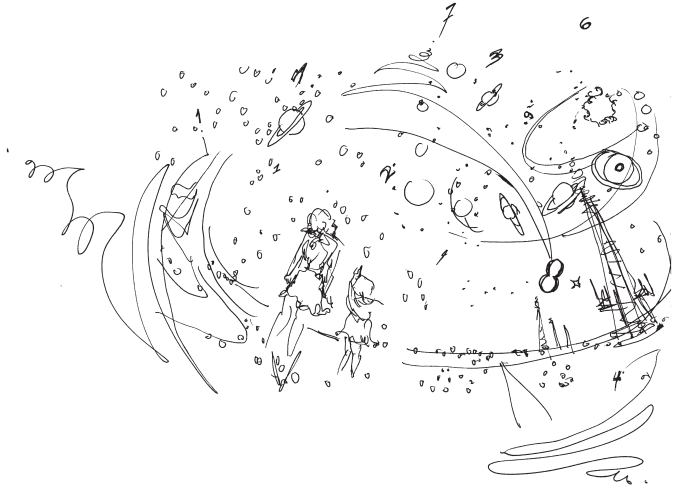
The

Numberverse

How numbers are bursting out of everything and just want to have fun



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For Mick Day, who taught me almost all of this without me noticing.

Foreword by Robin Ince

We are surrounded by numbers; numbers to persuade us, numbers to scare us, numbers to categorise us.

Political parties try to lure us into their way of thinking with big numbers that are meant to sum up wealth or sickness. Soap and yoghurt advertisements try to attract us with numbers that aim to persuade us our guts would be happier or our faces more beautiful if we purchase them.

We are numbers. We exist, knowingly or unknowingly – from our bank account to our national insurance number – as numbers in files, documents and programmes. I have no idea how many different numbers I am. Do you know how many you are? I am 45, I live at 81, my shoes are size 9 and I am often on the 1046.

Despite their omnipresence, numbers can still jar our mind and befuddle our judgement. They can make reality seem unreal.

Numbers are a language we use to explain the world – a language that confounds many of us, too. With the enormous enlightening and obfuscating fog of the mass media, the flood of information now available to us, it is a language we must be willing to interrogate if we are to understand ourselves and the world we live in.

Andrew Day offers the first steps to opening the minds of children to the excitement of numbers and why, through centuries of civilisation and human imagination, they have helped us to frame and shape our world.

These are the numbers that help us to understand why the universe is as it seems to be.

These are the numbers that are pored over by physicists watching bundles of particles colliding at speeds near that of light.

These are the numbers that tell us what page to turn to.

When I think of numbers I think of Douglas Adams and 42, the Answer to Life, the Universe and Everything.

I think of 150, Dunbar's number, the number of people you can maintain a stable social relationship with before your social brain starts to struggle.

I think of 12, the number of angry men in one of my favourite childhood films.

And I think of 23, the numbers of pairs of chromosomes that make us what we are, a quizzical creature that thrives on its own curiosity.

The delight of being a parent is that it gives you an alibi to interrogate the world in the guise of helping your child to learn. This act of educational altruism offers not merely the joyful reward of seeing a child comprehending the world, but also revivifying the adult's intrigue in it all too.

This book reminds us that it is not enough to teach the basics of mathematics. We also need to ask why maths exists and what a world without it might be like. I believe that by understanding why something exists or was created, we are further drawn into a subject. It is not just a case of knowing your times tables but understanding the needs that led to their creation.

The more we understand why, the more we are drawn into the adventure of learning.

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Introduction

How This Book Was Born

I didn't mean to get interested in maths. I wasn't even supposed to be doing it. What I normally do is visit schools and run philosophy sessions for an hour at a time. Doing philosophy with children doesn't mean telling kids the names of old dead men and what their theories were. We get children to philosophise themselves: to give reasons and compare ideas. We call these lessons enquiries. In our class enquiries we touch on, among other things, science, morals, religion, language … and sometimes numbers.

Then one day a school asked me to show their staff how to use the philosophical approach with all subjects – in other words, to turn lessons into enquiries, putting the children themselves at the heart of the lesson, with their curiosity driving it. This was daunting because I was training people more qualified and experienced than me. But at least I had an idea. So, with the help of some colleagues and the teachers themselves, I began to make some progress. That is how I ended up trying to create maths lessons.

Teachers are sometimes surprised to be told that philosophy and mathematics are closely related. They expect there to be links between philosophy and religion, psychology and literature, and there are, but philosophy has been practised by mathematicians, and vice versa, from the very beginning. In fact, Pythagoras, one of the earliest

mathematicians, and perhaps the easiest one to name, was also a philosopher in the sense we understand it now.

As a philosopher, I had a grasp of some of the fundamental questions of what mathematics is. Admittedly, having not studied maths to a very high level it was impossible for me to follow the expert answers to those questions. But, as I explained to some of the teachers I met, not being good at something is sometimes an ideal position from which to teach it, because you can empathise with the students, whereas if you find something easy, your hardest task can be to see why someone else would fail to understand.

When I sat down to start, my first thought was that you would need to make mathematics practical – to show how the things being taught are useful in everyday life. But, as I read the curriculum documents, I found constant references to children acquiring a sense of 'number' – in the abstract. They were supposed to gain an understanding of how the whole number system fitted together and the patterns that run through it. Good luck with that, I thought. I was sure children wouldn't be interested unless you found a practical application.

How wrong I was. I don't think I've ever been more wrong about anything.

As soon as I set youngsters puzzles using pure numbers they leapt into them. And what's more, they had questions. And theories. Sometimes they were well wide of the mark, but because I didn't mind, neither did they. Time after time, I saw children trying to make more sense of what they already knew, and to connect it up to new ideas. To them, the most elementary bits of mathematics were open to question and nothing was taken as read. I persevered with the practical aspect as well, because I thought it was important, but it often took more effort to engage the children that way.

Gradually, I started to understand how the way we run philosophy lessons can be a valuable tool in itself, and there have been moments where a whole class has suddenly been gripped by a question that they themselves have come up with: is zero divisible by anything? Is a square a rectangle? Is 7 a digit or a number? How many lines of symmetry does a square have? What about a circle? And I'm talking about children as young as 6, and in lower ability groups.

Why does it matter if moments like this arise? Well, first of all, we are always in a better position if children are *happy* to be doing maths, even occasionally. Whereas if they know there is no chance of them ever enjoying a maths lesson, they will switch off before it even starts.

I know what that feels like. I found maths dreary when I was a kid. We did maths in blue books (English was in warm, friendly orange or red) and whenever the stack of blue books came out, my heart sank. In maths, you got told about a thing, like long division, and then you did it over and over again in the hope that one day you would stop making mistakes. If you did stop making mistakes, you had to wait until other people stopped too before you could do something else.

That is probably slightly unfair on my teachers, but that is pretty much how I remember it. And you could rightly argue that things have got better since the early 1980s. However, there are still a lot of teachers out there who had the same negative experience as me, and they can't help passing some of that anti-maths feeling on to their own classes. Crucially, they learned to judge themselves as bad at (or bored by) maths very early on, and, as adults, the moment they get something wrong, or can't solve something, they are not intrigued but repelled.

Despite my complaints, I was very lucky because I have a few very different memories to draw on. Here is one …

Aged about 11, my class had two visitors from the local university one morning a week: a short bald man and a tall woman. They showed us

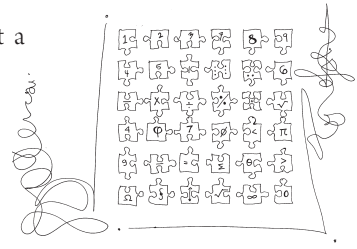
how to do algebra and draw graphs of the equations we made. We made up our own equations and challenged each other to solve them. It was pure fun, pure exploration. It was nothing like a maths lesson. When I did algebra in secondary school three years later I lapped it up, probably because I was biased in its favour.

Another oasis in the desert of boredom taught me something else: the value of a sense of purpose. It is the reason why the question, 'Miss, why are we doing this?', is actually a good one most of the time.

It was one morning with Mr Williams when we were 14. We had done trigonometry the year before, and I had hated it, completely alienated by the fact that we had to look up logarithms in the back of a little book. As far as I was concerned, if you were going to look stuff up in a book, why not just look up the answer! Those logarithms meant nothing to me, and I was the kind of kid who needed to know why something was done. Then one day Mr Williams showed us how you could, if you wanted, calculate any of those logarithms yourself. I don't remember how it was done (on the board, I remember, there was a circle cut into quarters and different quarters related to sine, cosine or tangent, and the fourth wasn't used) but remembering the details wasn't the point of the exercise. The point was that we now saw why we don't calculate those values each time – because it takes ages.

When I visit classrooms now, these two experiences influence what I want to bring in with me: the sense of *pleasure* we had when exploring algebra, and the sense of *purpose* I got from having trigonometry procedures explained.

With this in mind, I have looked at a number of mathematical topics, and wondered how to give people who study them both pleasure and purpose. And my best answers fall into



two categories: puzzles, which we do for pleasure, and problems, which we solve for a purpose.

A puzzle is something that tickles our brain and exercises it, but is not useful. Crosswords, for example, are of no value in the struggle to survive, and jigsaws are just time-consuming ways of making a picture that we could buy whole. But they are proof that we can enjoy exercising our brains just for the fun of it.

A problem, on the other hand, I would define as something that we feel driven to fix, even if that problem is an imaginary scenario. Examples of this would be how much interest a debt will rack up over a given period, or how many pounds you have to lose per week if you want to be half a stone thinner for your holiday.

The use that problems and puzzles have is that they are compelling. Once we see the problem, our minds can't help but try to solve it. It is just how we are. And that is the hook, the spark and trigger that a lesson needs. If children are guided, and not merely instructed, they will become adept at recognising problems and engaging with them.

It is really important to remember that children aren't born hating maths. They learn that hate. What they are born with is the impulse to enjoy exploring ideas and make sense of what they are doing. We just need to tap into it, and not trample on it because our eyes are fixed on assessments, streaming and objectives. I'm not against those three things by the way – I just want them not to spoil teaching.

Far too often, the need to simply get the answers right will take over. Then, children who can pick up new methods quickly will excel, and anyone sitting back to think about how it all works will just look slow-witted. In most classrooms, and among children where academic achievement is everything, the ability to do maths fast is tied to self-esteem. 'Brainy' children jealously guard their position as high-achievers, sometimes because it is the only source of self-esteem they have.

Consequently, many thoughtful children turn their backs on maths, emotionally speaking, because their approach and their needs are seen as inappropriate to maths, but that is nonsense. You can come at it from different angles and be good at it in different ways. That is not the same as saying that anyone can be good at maths or that anyone can reach any level. I like to compare it to athletics: every class will have a fastest runner and a slowest, but we can teach each of them to become a better runner, and with the right encouragement, some of the slower runners might still enjoy it the most.

What This Book Does

This book offers ways to make maths interesting, enjoyable and challenging. It does this in five main ways.

There are *stories or facts to interest teachers*. This is because the first stage in making maths interesting and enjoyable is for the teachers to feel that way about what they are teaching. Remember that by showing your interest you are modelling that interested attitude to your class. The stories and facts are not all intended for use in the classroom but some teachers may find ways to exploit them.

There are also ideas for how to *make maths fun*. Children can't have fun while learning all the time at school, but this truism is sometimes put forward by killjoys who argue that fun is educationally worthless. I hope you don't think that and won't be swayed by those who do. Most teachers try to scatter pieces of fun into their weekly timetable, and they will get some help here in putting it *into* the maths, not having fun *after* all the maths is done.

There are ways of *introducing new topics* in most chapters. The approach is to ask ourselves why this topic is worth studying, who

first developed the ideas and why. This links up to what I say above about a sense of purpose.

But this approach is not limited to introductions. It also works as a way of *opening up familiar topics*, both to a teacher who has had them on the curriculum for years, and to children who are revisiting an area studied to a certain depth in previous classes.

The book also tries to have an effect on the way the lesson is conducted. There are descriptions and explanations of *skills and habits* that make the classroom a place of active enquiry, not passive instruction.

I should make it clear at this early stage that this book doesn't attempt to show anyone how to teach the whole curriculum, or How to Teach, full stop. It is intended as a technique (or set of techniques) to make certain things happen in a classroom. There are other things that need to happen in the classroom which are not the subject of this book. What is more, there are various ways of getting the results we are after. When educational ideas get a bad name, it is often because grandiose claims have been made for what they achieve and how superior they are. The way of teaching I describe need not displace all other ways of teaching – just the ones that you have decided don't work as well.

How is the Book Meant to Be Used?

The chapters mostly have three parts. The first part is for the teacher to read, mainly for pleasure, but also to get a perspective on a particular topic. Sometimes the discussion is directly about teaching and in others I share my enthusiasm for the philosophy and history of mathematics in the hope that some of it will be contagious.

The second part of each chapter, called *Things to Do*, offers ideas for what to do in the classroom. I have highlighted the key instructions for the teacher like this:

Do Take seven boxes of twelve pens (or some similar sets of items) and show them to the class.

Say Who can work out how many pens there are altogether? You can open the boxes if you want!

I have kept these to a minimum so that a teacher preparing for a lesson can scan the page and focus on the main actions and instructions. It is unusual to tell teachers directly what to say, but the choice of language is really important, so I have tried to take away the difficulty of choosing the exact wording of instructions. I have had plenty of time to make these instructions as simple and clear as I can, hopefully saving you the trouble.

If the activities strike you as minimal, that is because each one is designed to generate discussion, prediction and a bit of healthy confusion. Your job as teacher is to facilitate the class's journey through all of this. You are not expected to concern yourself too much over what the children will conclude in their discussion, only that the discussion is focused and constructive – not meandering and tangential. As far as possible, I have tried to indicate the likely areas their conversation will cover. But I hope there will be plenty of surprises for you too!

The third part is a *Thing to Say* or a *Key Word*. These are practical tips on what to say to a class to create the right learning atmosphere, or words to describe important aspects of this style of teaching. Some of these key words – such as 'resilience' or 'discovery' – will be familiar to you.

I hope that reading all of each chapter in order will be satisfying, but also that you will find it enjoyable to dip in and out of, trying out

different lessons or just thinking about them. The reader can read all of every chapter, in the order they appear, but that is not what I have imagined you doing. You can easily skip to the chapters that sound most appealing or read through the Things to Do for lesson ideas.

At the beginning of the book are four or five chapters with very open topics, suiting a variety of ages and levels, without any predetermined content goal. The intention is for the discussion to stray across the maths curriculum. Later chapters pick out a few big curriculum topics.

The book deals with maths from its most basic levels upwards. It touches on more abstract, theoretical areas, such as different bases and irrational numbers, but I have deliberately avoided the mainstays of the secondary curriculum. That is because I think that primary-level mathematics (whether it is taught in primary school or later) is the key to everything: if they fall in love at that age, they will be set up for life.

What Difference Will the Book Make to the Children?

The purpose of many of the activities in this book is to make children feel stuck. I am trying to get them to feel comfortable with the 'stuck' feeling as a temporary but necessary stage on the way to success. I regard it as necessary because I want children to learn how to work their way out of it. That is why when children are stuck, it is important to wait with them while they detach their minds from the snag they have got caught on.

We need to be prepared to show real patience. Because if we are impatient about the learning process – as learners, parents or teachers – the

process will be incomplete and we end up with half-learned concepts, which we can't build on.

So, if you are going to use this book, what you should be looking for is moments where a child is fascinated, frustrated or foxed. Instead of trying to end or avoid this state by steering children to an answer, you should be trying to exploit it, by getting the child to reveal what is bugging her or asking her to suggest solutions and test them out. In this way, we are helping children to develop strategies to articulate and solve problems.

The benefits overlap and interconnect, but here are some viewpoints.

A sense of *ownership of the learning process* is important. Children want to understand what they are doing and why they are doing it. If there is a lesson objective on the board, it needs to be an objective that is meaningful to the children, not just 'what we are learning today'. Then the work – and its completion – will be intrinsically rewarding; that is, worth doing for its own sake. This book is always trying to grant the class this ownership by provoking their initial interest and training the teacher to capitalise on it.

Children are encouraged to *actively make sense of content* – they ask questions, reflect, experiment and try to generalise. They link what they encounter in the classroom with what they experience outside of it and elsewhere in the curriculum. This also relates to the comprehension and retention of new information. They will understand and remember what they find out in the lesson if they are allowed to piece it together in a way that makes sense to them.

Engagement with learning itself is a battle for teachers in some schools. The very idea of what school represents is a barrier with certain youngsters. So, it needs to be demonstrated to the children that they are answering questions that they themselves are moved to ask. The chil-

"A 'How To' guide with a difference. This book allowed me, as an experienced teacher, a fascinating insight into the world of maths and practical ways to help children truly understand the abstract nature of this subject. There is a glorious mixture of practical ideas, theory and anecdotes making this a very addictive read. I love the way it can be used as a dip-in-and-out planning resource or as an absorbing cover to cover read. It guides teachers gently but firmly towards being a facilitator of learning, showing us how to use enquiry to help pupils along the path of discovery."

Kirstin Fisher, Assistant Head Teacher, Ravenscroft Primary School

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Andrew Day is a writer of plays, films, stories and poems. He joined the Philosophy Foundation as quickly as he could when he found out they bring philosophy into schools. His degree in Philosophy and Social Anthropology has been invaluable in teaching various things to people of all ages, nationalities and backgrounds – from inner city nurseries to international banks.

"Andrew Day offers the first steps to opening the minds of children to the excitement of numbers and why, through centuries of civilisation and human imagination, they have helped us to frame and shape our world."

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